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Research Description: In the framework of continuum mechanics the motion of deforming bodies is described by systems of nonlinear evolution equations, that arise by combining balance laws with constitutive relations characterizing the material response. The nonlinear character of the material response often induces a destabilizing mechanism that competes with dissipative mechanisms, such as viscosity or thermal diffusion. As a result of the competition coherent structures may appear, which at some level of modeling manifest themselves as singularities in the solutions of the corresponding model. These structures are diverse in nature, ranging from shock waves to shear bands to propagating phase boundaries, and their understanding is critical in the design of numerical schemes. Various instances of such phenomena are studied as part of this project:

In the present reporting period our effort concentrated in the study of self-similar viscous and fluid-dynamic limits. The invariance of hyperbolic systems of conservation laws under dilations of coordinates is a key property underlying much of the current theory. Viscous perturbations introduce an additional parabolic scale and this invariance is lost. Understanding how the two scales interact for small viscosities is an important step in the process of studying viscous limits for general solutions. One illuminating step in that direction is to consider the Riemann problem and to study artificial regularizations, rigged so as to preserve the invariance under dilations of coordinates and the entropy structure of the system. It provides information on how viscosity regularizes the whole wave fan emerging from Riemann data. At the technical level, it leads to study of singular perturbations for non-autonomous boundary-value problems, and the limiting process involves variation estimates.

This approach was tested in the context of self-similar fluid dynamic limits for the Broadwell model with Riemann, Maxwellian data. This is a natural context to consider the effect of relaxation mechanisms on shock capturing. The questions under consideration is what admissibility restrictions are entailed on shocks by relaxation. It turns out that the limiting solutions, as the mean free path goes to zero, satisfy a pair of conservation laws and consist of two wave fans separated by a constant state. Each wave fan is associated with one of the characteristic fields and is either a rarefaction wave or a shock wave. The shocks satisfy the Lax shock conditions and have the internal structure of a Broadwell shock profile.

Technical Description A technical description of completed projects, supported by the Office of Naval Research under contract N00014-93-0015, follows.

[1] ***Wave structure induced by fluid dynamic limits in the Broadwell model.*** submitted to Arch. Rational Mech. Analysis.

We study the structure entailed by self-similar fluid dynamic limits in the context of Broadwell model. The resulting problem in the self-similar variable $\xi = x/t$ reads:

$$\begin{aligned} (-\xi+1)\, f_1' &= \frac{1}{\varepsilon}(f_3^2 - f_1 f_2) \\ (-\xi-1)\, f_2' &= \frac{1}{\varepsilon}(f_3^2 - f_1 f_2) \\ -\xi\, f_3' &= \frac{1}{2\varepsilon}(f_1 f_2 - f_3^2) \\ f_i(\pm\infty) &= f_{i,\pm}\,, i = 1,2,3 \end{aligned} \tag{B}$$

The data are Riemann and Maxwellian data, $f_{3,\pm}^2 - f_{1,\pm} f_{2,\pm} = 0$. The existence of solutions for the singular boundary value problem was studied by Slemrod and Tzavaras, who also show that any family of solutions of (B) is of bounded variation and that any limiting point f of such a family satisfies the Riemann problem for

$$\begin{aligned} \frac{\partial}{\partial t}\Big(f_1 + f_2 + 4(f_1 f_2)^{1/2}\Big) + \frac{\partial}{\partial x}(f_1 - f_2) = 0\,, \\ \frac{\partial}{\partial t}(f_1 - f_2) + \frac{\partial}{\partial x}(f_1 + f_2) = 0\,. \end{aligned} \tag{FE}$$

(FE) is a pair of conservation laws in the variables $\rho = f_1 + f_2 + 4(f_1 f_2)^{1/2}$ and $\rho u = f_1 - f_2$. We take up the problem of the structure for the limiting solution f. First, certain representation formulas expressing the rate of collisions $Q(f^\varepsilon)/\varepsilon$ as an averaging process are derived. It is then possible to translate the structure problem into studying properties of a measure ν (the weak-star limit of $Q(f^\varepsilon)/\varepsilon$), which incorporates the form of the limiting

f and its support coincides with the set where f is nonconstant. The points where f is nonconstant can be grouped into two wave fans and are either points of shocks or points of rarefactions. Moreover, a function g related to the antiderivatives of the eigenvalues is maximized at such points. This plays the role of an admissibility restriction and yields the Lax shock conditions at discontinuities. Finally, at shocks the family f^ϵ has the internal structure of a Broadwell shock profile.